Model description:
Cortical Microcircuit (Potjans & Diesmann, 2014)

source: https://github.com/INM-6/microcircuit-PD14-model

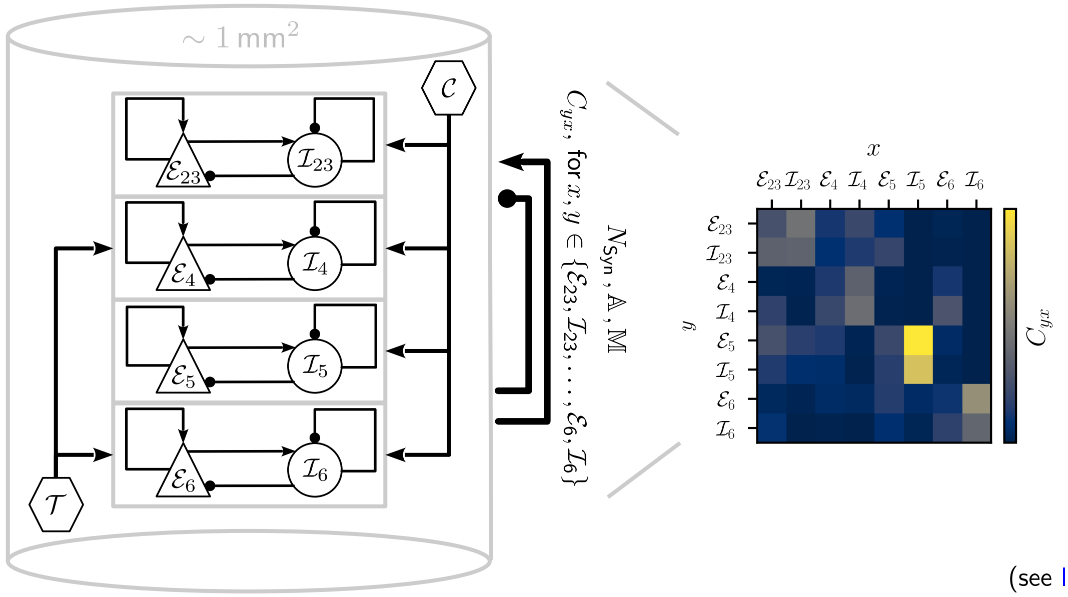
last update: March 24, 2026

This document contains a detailed, mathematical description of the Cortical Microcircuit model, initially developed by Potjans & Diesmann (2014). It does not contain any information on specific numerical implementations. The description follows the guidelines of Nordlie et al. (2009).

Contents

1 Model description

Summary	
Populations	8 cortical populations in 4 layers (L2/3, L4, L5, L6), driven by cortico-cortical inputs ($\mathcal{C}$) and a thalamic population ($\mathcal{T}$)
Connectivity	random, independent, population specific
Neuron model	cortex: leaky integrate-and-fire (LIF); cortico-cortical inputs: constant currents (DC); thalamus: Poisson point process
Synapse model	exponential postsynaptic currents with static, normally distributed weights and delays
Predictions	population specific spiking activity



(see legend)

Populations		
Name	**Elements**	**Size**
local cortical populations $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$	LIF	N_x
total local cortical population $\mathcal{P} = \bigcup_x x$	LIF	$N = \sum_x N_x$ (see remark 1)
non-local cortical populations $\mathcal{C}_x$ (cortico-cortical inputs)	constant currents (DC)	N_x
total population of non-local cortical neurons $\mathcal{C} = \bigcup_x \mathcal{C}_x$	constant currents (DC)	$N = \sum_x N_x$
thalamic neurons $\mathcal{T}$	realizations of Poisson point process	$N_{\mathcal{T}}$

Table 1: Description of the network model (continued on next page).

Connectivity		
Source	**Target**	**Pattern**
$x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	• random, fixed total number Q_{yx} of connections[1] (see remark 1) • synaptic weights J_{ij} $(\forall i \in y, j \in x)$ • spike-transmission delays d_{ij} $(\forall i \in y, j \in x)$
$\mathcal{T}$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	• random, fixed total number $Q_{y\mathcal{T}}$ of connections[1] • synaptic weights J_{ij} $(\forall i \in y, j \in \mathcal{T})$ • spike-transmission delays d_{ij} $(\forall i \in y, j \in \mathcal{T})$
$\mathcal{C}_y$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	• one-to-one[2]

Connectivity patterns:

[1] *random, fixed total number* (N_{Syn}): This rule establishes a total number of

$$Q_{yx} = \frac{\ln\left(1 - C_{yx}\right)}{\ln\left(1 - \left(N_x N_y\right)^{-1}\right)}, \tag{1}$$

connections between a source population x of size N_x and a target population y of size N_y. C_{yx} denotes the connection probability. Sources and targets are randomly and independently drawn from x and y with replacement. Multiple connections between two neurons and self-connections are permitted ($\mathbb{M}$, $\mathbb{A}$).

[2] *one-to-one* (δ): Each neuron in the source population is connected to one corresponding neuron in the target population (bijection).

(see "Network sketch" above and Senk et al., 2022)

Table 1: Description of the network model (continued).

Neurons	
Local cortical neurons	
Type	leaky integrate-and-fire (LIF)
Description	dynamics of membrane potential $V_i(t)$ and spiking activity $s_i(t)$ of neuron $i \in x$ for $x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$: • emission of kth $(k = 1, 2, \ldots)$ spike of neuron i at time t_i^k if $$V_i\left(t_i^k\right) \geq \theta \qquad (2)$$ with spike threshold θ • spike train $s_i(t) = \sum_k \delta(t - t_i^k)$ • reset and refractoriness: $$\forall k,\ \forall t \in \left[t_i^k,\, t_i^k + \tau_{\mathsf{ref}}\right]: \quad V_i(t) = V_{\mathsf{reset}} \qquad (3)$$ with refractory period τ_{ref} and reset potential V_{reset} • subthreshold dynamics of membrane potential $V_i(t)$: $$\begin{aligned} &\forall k,\ \forall t \notin \left[t_i^k,\, t_i^k + \tau_{\mathsf{ref}}\right]: \\ &\tau_{\mathsf{m}} \frac{\mathsf{d}V_i(t)}{\mathsf{d}t} = \left[E_{\mathsf{L}} - V_i(t)\right] + R_{\mathsf{m}} I_i(t) + R_{\mathsf{m}} I_{i,\mathcal{C}_y} \end{aligned} \qquad (4)$$ with membrane time constant τ_{m}, membrane resistance R_{m}, resting potential E_{L}, total synaptic input current $I_i(t)$ (see "Synapses"), and constant background current $I_{i,\mathcal{C}_y} = I_{\mathcal{C}_y}$ for $i \in y$ (see "Cortico-cortical inputs") (see App. A1 for the normal form of the combined subthreshold neuron-synapse dynamics)
Non-local cortical neurons (cortico-cortical inputs)	
Type	constant (direct) currents (DC)
Description	population specific constant input current of magnitude $$I_{\mathcal{C}_x} = K_{\mathcal{C}_x} \cdot I_{\mathcal{C}} \quad \left(\forall x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}\right),$$ with cortico-cortical in-degree $K_{\mathcal{C}_x}$, and mean current $$I_{\mathcal{C}} = \bar{I} \cdot \tau_{\mathsf{s}} \cdot \nu_{\mathcal{C}}$$ generated by single cortico-cortical input spike train with rate $\nu_{\mathcal{C}}$, convolved with an exponential kernel with amplitude $\bar{I}$ and time constant τ_{s} (see remark 2)
Thalamic neurons	
Type	Poisson point process
Description	spike trains $s_i(t)$ $(i \in \mathcal{T})$ modeled as independent realizations of Poisson point process with piece-wise constant rate $$\nu_{\mathcal{T}}(t) = \begin{cases} 0 & t < t_{\mathsf{start}} \\ \nu_{\mathcal{T}} & t_{\mathsf{start}} \leq t < t_{\mathsf{stop}} \\ 0 & t \geq t_{\mathsf{stop}} \end{cases} \qquad (5)$$

Table 2: Description of the network model (continued).

Synapses	
Type	exponential postsynaptic currents with static weights and delays
Description	• total synaptic input current $I_i(t)$ to neuron i ($\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$) is governed by: $$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{\tau_\mathrm{s}}\right) I_i(t) = f_i(t) \quad (6)$$ with superposition from all neurons $j \in x,\ \forall x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}\}$ $$f_i(t) = \sum_x \sum_j f_{ij}(t) = \sum_x \sum_j \hat{I}_{ij} s_j(t - d_{ij}) \quad (7)$$ of weighted spike trains with static synaptic weights $\hat{I}_{ij}$, synaptic time constant τ_s, and spike transmission delays d_{ij} (see App. A1 for the normal form of the combined subthreshold neuron-synapse dynamics) • solution of (6) for $f_{ij}(t) = \hat{I}_{ij} s_j(t)$ and $I_{ij}(t = 0) = 0$: $$\mathrm{PSC}_{ij}(t) = \hat{I}_{ij} \exp(-t/\tau_\mathrm{s}) \Theta(t) \quad (8)$$ with Heaviside function $\Theta(\cdot)$ ↷ (exponential decaying) postsynaptic current triggered by a single presynaptic spike • solution of (4) for $I_i(t) = \mathrm{PSC}_{ij}(t)$, $V_i(t = 0) = 0$, and $E_\mathrm{L} = 0$: $$\mathrm{PSP}_{ij}(t) = \hat{I}_{ij} R_\mathrm{m} \frac{\tau_\mathrm{s}}{\tau_\mathrm{s} - \tau_\mathrm{m}} \left(e^{-t/\tau_\mathrm{s}} - e^{-t/\tau_\mathrm{m}}\right) \Theta(t) \quad (9)$$ • PSC amplitude (synaptic weight): $$\hat{I}_{ij} = \frac{J_{ij}}{J_\mathrm{unit}(\tau_\mathrm{m}, \tau_\mathrm{s}, R_\mathrm{m})} \quad (10)$$ parameterized by PSP amplitude $J_{ij} = \max_t\left(\mathrm{PSP}_{ij}(t)\right)$ with unit PSP amplitude (PSP amplitude for $\hat{I}_{ij} = 1$): $$J_\mathrm{unit}(\tau_\mathrm{m}, \tau_\mathrm{s}, R_\mathrm{m}) = R_\mathrm{m} \frac{\tau_\mathrm{s}}{\tau_\mathrm{s} - \tau_\mathrm{m}} \left(\left[\frac{\tau_\mathrm{m}}{\tau_\mathrm{s}}\right]^{-\tau_\mathrm{m}/(\tau_\mathrm{m} - \tau_\mathrm{s})} - \left[\frac{\tau_\mathrm{m}}{\tau_\mathrm{s}}\right]^{-\tau_\mathrm{s}/(\tau_\mathrm{m} - \tau_\mathrm{s})} \right) \quad (11)$$ and time to PSP maximum: $$t_\mathrm{max} = \frac{\tau_\mathrm{s}\tau_\mathrm{m}}{\tau_\mathrm{m} - \tau_\mathrm{s}} \ln\left(\frac{\tau_\mathrm{m}}{\tau_\mathrm{s}}\right) \quad (12)$$

Table 3: Description of the network model (continued).

Synapses (continued)	
Description	• synaptic weights $$\hat{I}_{ij} = \begin{cases} \mathsf{max}(0, z_{yx}), & j \in x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}\} \\ \mathsf{min}(0, z_{yx}), & j \in x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\} \\ \bar{I}_{yx}, & j \in x = \mathcal{C}_y \\ 0 & \text{if connection } j \to i \text{ does not exist} \end{cases} \quad (13)$$ with $$z_{yx} \sim \mathcal{N}\left\{\bar{I}_{yx},\, \sigma^2_{\mathsf{s},yx}\right\} \quad (14)$$ drawn from a normal distribution with mean $\bar{I}_{yx}$, variance $\sigma^2_{\mathsf{s},yx}$ note: clipping of synaptic weights leads to a deviation of the total number of synapses with non-zero weights from K_{yx} (see "Connectivity") • distributed synaptic delays $$d_{ij} = \begin{cases} \mathsf{max}(d_{\mathsf{min}}, z_{yx}), & \forall i,\ \forall j \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}\} \\ \bar{d}_{yx}, & \forall i,\ \forall j \in x = \mathcal{C}_y \end{cases} \quad (15)$$ with $$z_{yx} \sim \mathcal{N}\left\{\bar{d}_{yx},\, \sigma^2_{\mathsf{d},yx}\right\} \quad (16)$$ drawn from a normal distribution with mean $\bar{d}_{yx}$, variance $\sigma^2_{\mathsf{d},yx}$, and minimal delay d_{min}

Initial conditions	
Type	random initial membrane potentials and homogeneous initial synaptic currents
Description	• membrane potentials $$V_i(t=0) \sim \mathcal{N}(\bar{V}_{0,x}, \sigma^2_{\mathsf{v},x}) \quad (17)$$ randomly and independently drawn from a normal distribution with mean $\bar{V}_{0,x}$ and variance $\sigma^2_{\mathsf{v},x}$ ($\forall i \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$; see remark 4) • synaptic currents: $I_i(t=0) = 0\,\mathsf{pA}$ ($\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$)

Table 4: Description of the network model (continued).

2 Model parameters

Network and connectivity

Population sizes

x	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$	$\mathcal{T}$
N_x	$20,683$	$5,834$	$21,915$	$5,479$	$4,850$	$1,065$	$14,395$	$2,948$	902

Connection probabilities C_{yx}

y \ x	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$	$\mathcal{T}$
$\mathcal{E}_{23}$	0.1009	0.1689	0.0437	0.0818	0.0323	0.0	0.0076	0.0	0.0
$\mathcal{I}_{23}$	0.1346	0.1371	0.0316	0.0515	0.0755	0.0	0.0042	0.0	0.0
$\mathcal{E}_4$	0.0077	0.0059	0.0497	0.1350	0.0067	0.0003	0.0453	0.0	0.0983
$\mathcal{I}_4$	0.0691	0.0029	0.0794	0.1597	0.0033	0.0	0.1057	0.0	0.0619
$\mathcal{E}_5$	0.1004	0.0622	0.0505	0.0057	0.0831	0.3726	0.0204	0.0	0.0
$\mathcal{I}_5$	0.0548	0.0269	0.0257	0.0022	0.0600	0.3158	0.0086	0.0	0.0
$\mathcal{E}_6$	0.0156	0.0066	0.0211	0.0166	0.0572	0.0197	0.0396	0.2252	0.0512
$\mathcal{I}_6$	0.0364	0.0010	0.0034	0.0005	0.0277	0.0080	0.0658	0.1443	0.0196

Neuron

Name	**Value**	**Description**
Local cortical neurons		
θ	$-50\,\mathrm{mV}$	spike threshold
E_L	$-65\,\mathrm{mV}$	resting potential
τ_m	$10\,\mathrm{ms}$	membrane time constant
C_m	$250\,\mathrm{pF}$	membrane capacitance
R_m	$\tau_\mathrm{m}/C_\mathrm{m} = 40\,\mathrm{M\Omega}$	membrane resistance
V_reset	$-65\,\mathrm{mV}$	reset potential
τ_ref	$2\,\mathrm{ms}$	absolute refractory period
Non-local cortical neurons (cortico-cortical inputs)		
$\nu_\mathcal{C}$	$8\,\mathrm{s}^{-1}$	rate of a single cortico-cortical input
$I_\mathcal{C}$	$\bar{I}_\mathcal{C}\tau_\mathrm{s}\nu_\mathcal{C} = 0.3\,\mathrm{pA}$	mean current generated by a single cortico-cortical input

Population specific cortico-cortical in-degrees $K_{\mathcal{C}_x}$

$\mathcal{C}_y$	$\mathcal{C}_{\mathcal{E}_{23}}$	$\mathcal{C}_{\mathcal{I}_{23}}$	$\mathcal{C}_{\mathcal{E}_4}$	$\mathcal{C}_{\mathcal{I}_4}$	$\mathcal{C}_{\mathcal{E}_5}$	$\mathcal{C}_{\mathcal{I}_5}$	$\mathcal{C}_{\mathcal{E}_6}$	$\mathcal{C}_{\mathcal{I}_6}$	$\mathcal{C}_\mathcal{T}$
$K_{y\mathcal{C}_y}$	1600	1500	2100	1900	2000	1900	2900	2100	–

Thalamic neurons		
$\nu_\mathcal{T}$	$120\,\mathrm{s}^{-1}$	rate of thalamic neurons
t_start	$700\,\mathrm{ms}$	start time of thalamic input
$\Delta t_\mathcal{T}$	$10\,\mathrm{ms}$	duration of thalamic input
t_stop	$t_\mathrm{start}+\Delta t_\mathcal{T} = 710\,\mathrm{ms}$	stop time of thalamic input

Table 5: Model parameters. Parameters derived from other parameters are marked in gray (continued on next page).

Synapse		
Name	**Value**	**Description**
J	$0.15\,\mathrm{mV}$	(mean) weight (PSP amplitude) of excitatory synapses
$\bar{I}_{yx}$		synaptic weights
	$J/J_{\mathsf{unit}} \approx 87.81\,\mathrm{pA}$	$\forall x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}_y\}$, $\forall y \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$, except for:
	$2J/J_{\mathsf{unit}}$	$(x, y) = (\mathcal{E}_{23}, \mathcal{E}_4)$
	$-4J/J_{\mathsf{unit}}$	$\forall x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$, $\forall y \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$
$\sigma_{\mathsf{s},yx}$	$0.1 \cdot \bar{I}_{yx}$	standard deviation of weight distribution
τ_{s}	$0.5\,\mathrm{ms}$	synaptic time constant
$\bar{d}_{yx}$		mean spike transmission delays
	$1.5\,\mathrm{ms}$	$x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}_y\}$, $y \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$
	$0.75\,\mathrm{ms}$	$x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$, $y \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$
$\sigma_{\mathsf{d},yx}$	$0.5 \cdot \bar{d}_{yx}$	standard deviation of spike transmission delays
d_{min}	$0.1\,\mathrm{ms}$	minimal spike transmission delay

Initial conditions

Population specific mean and standard deviation of initial membrane-potential distributions

population x	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$
$\bar{V}_{0,x}$ (mV)	-68.28	-63.16	-63.33	-63.45	-63.11	-61.66	-66.72	-61.45
$\sigma_{\mathsf{v},x}$ (mV)	5.36	4.57	4.74	4.94	4.94	4.55	5.46	4.48

Table 6: Model parameters. Parameters derived from other parameters are marked in gray (continued).

Appendix

A1 Single-neuron dynamics in normal form (subthreshold)

- linear, inhomogeneous dynamics of synaptic input currents and (subthreshold) membrane potential for neuron $i \in y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$ (cf. eqs. (4) and (6)):

$$\begin{aligned} \dot{I}_i + \frac{1}{\tau_\mathsf{s}} I_i &= f_i(t) \\ \dot{V}_i + \frac{1}{\tau_\mathsf{m}} \Big[V_i - E_\mathsf{L} \Big] - \frac{R_\mathsf{m}}{\tau_\mathsf{m}} I_i &= 0 \end{aligned} \tag{18}$$

  with

$$f_i(t) = \sum_x \sum_{j \in x} \hat{I}_{ij} s_j(t - d_{ij}) \qquad (x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6, \mathcal{T}\}) \tag{19}$$

- rescale membrane potential $v_i(t) = V_i(t) - E_\mathsf{L}$ and total current $x_i(t) = \frac{R_\mathsf{m}}{\tau_\mathsf{m}} I_i(t)$:

$$\begin{aligned} \dot{x}_i + \frac{1}{\tau_\mathsf{s}} x_i &= \frac{R_\mathsf{m}}{\tau_\mathsf{m}} f_i(t) \\ \dot{v}_i + \frac{1}{\tau_\mathsf{m}} v_i - x_i &= 0 \end{aligned} \tag{20}$$

- normal form of neuron-i dynamics (20):

$$\frac{\mathsf{d}}{\mathsf{d}t} \boldsymbol{y}_i = \boldsymbol{A} \boldsymbol{y}_i + \boldsymbol{f}_i(t) \tag{21}$$

  with $D = 2$ dimensional state vector

$$\boldsymbol{y}_i(t) = \Big(x_i(t) \, , \, v_i(t) \Big)^\mathsf{T} , \tag{22}$$

  with constant $(D \times D)$ matrix

$$\boldsymbol{A} = \begin{bmatrix} -1/\tau_\mathsf{s} & 0 \\ 1 & -1/\tau_\mathsf{m} \end{bmatrix} , \tag{23}$$

  and inhomogeneity vector

$$\boldsymbol{f}_i(t) = \left(\frac{R_\mathsf{m}}{\tau_\mathsf{m}} f_i(t) \, , \, 0 \right)^\mathsf{T} \tag{24}$$

  (see Sec. 3.2.2 in Rotter & Diesmann, 1999)

- see App. A2 for an efficient, exact integration scheme of (21)
- back-transform to physical quantities:

$$\begin{aligned} V_i(t) &= v_i(t) + E_\mathsf{L} \\ I_i(t) &= \frac{\tau_\mathsf{m}}{R_\mathsf{m}} x_i(t) \end{aligned} \tag{25}$$

A2 Exact integration of single-neuron dynamics (subthreshold)

- exact integration of (21) for spikes arriving at the target neuron i on a time grid $\mathcal{T}_\Delta = \{t_k = k\Delta | k \in \mathbb{N}, \Delta \in \mathbb{R}^+\}$, i.e., for spike trains $s_j(t) = \sum_l \delta(t - t_j^l)$ with $t_j^l \in \mathcal{T}_\Delta$ (Rotter & Diesmann, 1999):

$$\boldsymbol{y}_i(t_{k+1}) = \boldsymbol{P} \boldsymbol{y}_i(t_k) + \boldsymbol{f}_i(t_{k+1}) \tag{26}$$

  with $(D \times D)$ propagator matrix (matrix exponential)

$$\boldsymbol{P} = e^{\boldsymbol{A}\Delta} \tag{27}$$

  with components

$$\boldsymbol{P} = \begin{bmatrix} e^{-\Delta/\tau_\mathsf{s}} & 0 \\ \frac{e^{-\Delta/\tau_\mathsf{m}} - e^{-\Delta/\tau_\mathsf{s}}}{1/\tau_\mathsf{s} - 1/\tau_\mathsf{m}} & e^{-\Delta/\tau_\mathsf{m}} \end{bmatrix} \tag{28}$$

  (see Sec. 3.2.2 in Rotter & Diesmann, 1999)

A3 Remarks

1. In the PyNEST implementation of the model provided here, the model size can be configured by the parameters `N_scaling` and `K_scaling`, which scale the number of neurons in the network and the number of synapses per neuron (in-degree), respectively. The original (default) full-scale model corresponds to `N_scaling=1` and `K_scaling=1`. Downscaling the model enables running it in the presence of limited computing resources (e.g., on a laptop). The scaling of the synapse number affects both the connections within the local network and the external (cortico-cortical) inputs. Without any compensation, changing the number of synapses would change the statistics of the total input current

$$I_i(t) = \sum_{x\in\{\mathcal{E}_{23},\ldots,\mathcal{I}_6\}} \sum_{j\in x} (s_j * \mathsf{PSC}_{ij})(t - d_{ij}) + I_{i,\mathcal{C}} \tag{29}$$

for each postsynaptic neuron i in population $y \in \{\mathcal{E}_{23},\ldots,\mathcal{I}_6\}$, and hence the firing statistics. In the PyNEST implementation provided here, this change in the activity statistics is suppressed by rescaling the average synaptic weights $\bar{I}_{yx}$ and injecting an additional constant input current into each neuron. In the following, we outline this compensation procedure.

Under the assumption that the spike trains $s_j(t)$ can be described by stationary Poisson point processes with rates ν_j, the mean and the variance of the total synaptic input current eq. (29) read[1]

$$\mu_i = \tau_{\mathsf{s}} \sum_{x\in\{\mathcal{E}_{23},\ldots,\mathcal{I}_6,\mathcal{C}_y\}} \sum_j \hat{I}_{ij}\nu_j \quad \text{and} \quad \sigma_i^2 = \frac{1}{2}\tau_{\mathsf{s}} \sum_x \sum_j \hat{I}_{ij}^2 \nu_j. \tag{30}$$

If we further assume that the variance $\sigma^2_{\mathsf{s},yx}$ of the synaptic weight distribution can be neglected, the population averaged mean and variance are approximated by

$$\mu_y = \langle \mu_i \rangle_{i\in y} = \tau_{\mathsf{s}} \sum_x K_{yx}\bar{I}_{yx}\nu_x \quad \text{and} \quad \sigma_y^2 = \langle \sigma_i^2 \rangle_{i\in y} = \frac{1}{2}\tau_{\mathsf{s}} \sum_x K_{yx}\bar{I}_{yx}^2\nu_x. \tag{31}$$

Here, K_{yx} refers to the average number of inputs a neuron in population y receives from the presynaptic population x, and ν_x to the population averaged firing rate of neurons in population x. Without further compensation, scaling the in-degrees $K^*_{yx} = fK_{yx}$ by some factor f would evidently change μ_y and σ_y^2. However, if we rescale the synaptic weights according to

$$\bar{I}^*_{yx} = \frac{\bar{I}_{yx}}{\sqrt{f}}, \tag{32}$$

the variance of the total input current is remains unchanged:

$$(\sigma_y^*)^2 = \frac{1}{2}\tau_{\mathsf{s}} \sum_x K^*_{yx}\bar{I}^{*,2}_{yx}\nu_x = \frac{1}{2}\tau_{\mathsf{s}} \sum_x fK_{yx}\left(\frac{\bar{I}^*_{yx}}{\sqrt{f}}\right)^2 \nu_x = \sigma_y^2. \tag{33}$$

If we furthermore inject an additional constant input current with amplitude

$$\Delta\mu_y = \left(1 - \sqrt{f}\right)\mu_y \tag{34}$$

into all neurons in population y, we simultaneously preserve the mean of the total input current:

$$\mu_y^* = \tau_{\mathsf{s}} \sum_x K^*_{yx}\bar{I}^*_{yx}\nu_x + \Delta\mu_y = \tau_{\mathsf{s}} \sum_x fK_{yx}\frac{\bar{I}_{yx}}{\sqrt{f}}\nu_x + \Delta\mu_y = \sqrt{f}\mu_y + \left(1 - \sqrt{f}\right)\mu_y = \mu_y. \tag{35}$$

(see remark 3). For more details and other scaling methods, see (van Albada et al., 2015).

2. In the original model of Potjans & Diesmann (2014), the cortico-cortical inputs to neurons in population y are modeled as $K_{y\mathcal{C}_y}$ independent realizations $s_j(t)$ ($j \in \mathcal{C}_y,\ y \in \{\mathcal{E}_{23},\ldots,\mathcal{I}_6\}$) of a Poisson point process with constant rate $\nu_{\mathcal{C}_y}$, filtered by an exponential kernel $\mathsf{PSC}(t)$ with time constant τ_{s} and amplitude $\bar{I}_{y\mathcal{C}_y}$. In the implementation provided here, these Poissonian inputs are replaced by constant external currents (DC). DC inputs are computationally less expensive, exactly reproducible, and lead to similar network activity statistics. When replacing cortico-cortical input spikes $s_j(t)$ by DC inputs, the current implementation preserves the mean input current

$$I_{\mathcal{C}_y} = \left\langle \sum_{j=1}^{K_{y\mathcal{C}_y}} (s_j * \mathsf{PSC})(t) \right\rangle = K_{y\mathcal{C}_y} \cdot \tau_{\mathsf{s}} \cdot \bar{I}_{y\mathcal{C}_y} \cdot \nu_{\mathcal{C}_y}. \tag{36}$$

[1] Here, we have absorbed the constant cortico-cortical input current $I_{i,\mathcal{C}}$ into the sum across all presynaptic sources.

3. In the scaled version of the model, the neurons receive –in addition to the cortico-cortical inputs– an additional constant input current of magnitude $\Delta\mu_y = (1-\sqrt{f})\mu_y$ (see remark 1). The total DC input current injected into neurons of population y is hence given by

$$\begin{aligned} I_{\text{DC},y} &= I^*_{y\mathcal{C}_y} + \left(1-\sqrt{f}\right)\mu_y \\ &= \sqrt{f} I_{y\mathcal{C}_y} + \left(1-\sqrt{f}\right)\left(\mu_{y,\text{loc}} + I_{y\mathcal{C}_y}\right) \\ &= I_{y\mathcal{C}_y} + \left(1-\sqrt{f}\right)\mu_{y,\text{loc}}, \end{aligned} \tag{37}$$

where $I^*_{y\mathcal{C}_y} = \tau_\text{s} \cdot f K_{y\mathcal{C}_y} \cdot \bar{I}_{y\mathcal{C}_y}/\sqrt{f} = \sqrt{f} I_{y\mathcal{C}_y}$ denotes the rescaled cortico-cortical input current, and $\mu_{y,\text{loc}} = \mu_y - I_{y\mathcal{C}_y}$ the mean input current resulting from synaptic inputs from the local network. In the absence of any initial spikes, the neurons in the network can generate spikes only if $I_{\text{DC},y}$ exceeds the rheobase current

$$I_{\text{rh},y} = \frac{\theta - E_\text{L}}{R_\text{m}}, \tag{38}$$

at least for some of the populations y. To ensure that population y of the scaled network becomes is activated by the external inputs, the scaling factor f must fulfill

$$I_{y\mathcal{C}_y} + \left(1-\sqrt{f}\right)\mu_{y,\text{loc}} \geq I_{\text{rh},y} \quad \Leftrightarrow \quad f \begin{cases} \geq f^*_y & \text{if } \mu_{y,\text{loc}} < 0 \\ \leq f^*_y & \text{if } \mu_{y,\text{loc}} > 0 \end{cases}, \tag{39}$$

with the critical scaling factor

$$f^*_y = \left(1 - \frac{I_{\text{rh},y} - I_{y\mathcal{C}_y}}{\mu_{y,\text{loc}}}\right)^2. \tag{40}$$

Typically, the mean of the local input current is negative, $\mu_{y,\text{loc}} < 0$, due to the dominant inhibitory feedback in balanced networks. In this case, the critical scaling factor f^*_y defines a lower bound for f. If the network is downscaled too much such that $f < f^*_y$ for some population y, this population can only be activated by other populations y' where $f > f^*_{y'}$.

4. The original model of Potjans & Diesmann (2014) uses *population unspecific* normal distributions of initial membrane potentials. By default, the current implementation uses *population specific* initial membrane potential distributions instead to speed up convergence to the stationary state. In the reference implementation provided here, the type of initial conditions can be set by the parameter `V0_type` (`"optimized"` [default] or `"original"`). In (Senk et al., 2026), the population specific initial conditions are referred to as *amended* initial conditions.

References

van Albada, S.J, Helias, M., Diesmann, M. (2015). Scalability of asynchronous networks is limited by one-to-one mapping between effective connectivity and correlations. *PLoS Computational Biology* 11(9):e1004490. https://doi.org/10.1371/journal.pcbi.1004490

Nordlie, E., Gewaltig, M.O., Plesser, H.E. (2009). Towards reproducible descriptions of neuronal network models. *PLoS Computational Biology* 5(8):e1000456. https://doi.org/10.1371/journal.pcbi.1000456

Potjans, T. C., Diesmann, M. (2014). The cell-type specific cortical microcircuit: relating structure and activity in a full-scale spiking network model. *Cerebral Cortex* 24(3):785–806. https://doi.org/10.1093/cercor/bhs358

Rotter, S., and Diesmann, M. (1999). Exact digital simulation of time-invariant linear systems with applications to neural modeling. *Biological Cybernetics* 81:381–402. https://doi.org/10.1007/s004220050570

Senk, J., Kriener, B., Djurfeldt, M., Voges, N., Jiang, H.-J., Schüttler, L., Gramelsberger, G., Diesmann, M., Plesser, H.E., van Albada, S.J. (2022). Connectivity concepts in neuronal network modeling. *PLoS Computational Biology* 18(9):e1010086. https://doi.org/10.1371/journal.pcbi.1010086

Senk, J., Kurth, A., Furber, S., Gemmeke, T., Golosio, B., Heittmann, A., Knight, J. C., Müller, E., Noll, T., Nowotny, T., Peraza Coppola, G., Peres, L., Rhodes, O., Rowley, A., Schemmel, J., Stadtmann, T., Tetzlaff, T., Tiddia, G., van Albada, S. J., Villamar, J., & Diesmann, M. (2026). Constructive community race: full-density spiking neural network model drives neuromorphic computing. *Neuromorphic Computing and Engineering* 6(1):012001. http://dx.doi.org/10.1088/2634-4386/ae379a